MATHEMATICAL INTIMACY WITHIN BLENDED AND FACE-TO-FACE LEARNING ENVIRONMENTS

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Oftentimes, learning mathematics involves using technology as a conscious attempt to understand the material in a new and accessible mode, to increase performance, and to expand one's knowledge regardless of the level of training. While comparative assessments of students' performances in blended and face-to-face environments are essential, the authors of this paper analyze students' mathematical intimacy and flow experiences, as well as their confidence and perseverance while learning mathematics in two different settings. Are students more engaged in problem solving, more inclined to experience joy, excitement and affection, more confident, and more persistent while doing mathematics in blended or in traditional learning environments? The authors of this paper aim to answer the question from the perspective of using MyMathLab, a Pearson based online course, within a blended teaching and learning environment for Algebra and Trigonometry, a large first year university mathematics course. This paper aims to analyze and interpret students' mathematical intimacy, confidence and perseverance in these two different learning environments.

MyMathLab is an online course designed by Pearson Education Canada for its Algebra and Trigonometry textbook. MyMathLab is built on the MathXL platform, Pearson's online homework and assessment system and is accessed via CourseCompass, the Pearson online learning environment. University professors can choose MyMathLab for teaching the whole math course or just some of its sections. MyMathLab offers instructors and students a remarkable selection of course materials that range from a large database of exercises to multimedia resources, such as video lectures, animations, and an electronic version of the textbook. Instructors are not constrained to draw on the existing database; rather new items can be added. Practice exercises regenerate automatically for an indefinite number of times, thus offering students the opportunity to rehearse each math problem. To aid comprehension of mathematics concepts students can use the interactive solution guide and worked examples accompanying each exercise in the database. Students receive instant feedback upon solving each exercise. Moreover, MyMathLab offers an online grade book, which automatically registers students' homework results and gives instructors control over computing final marks.

Implementing new technologies is especially significant at the undergraduate level where students encounter a wide range of definitions, theorems and proofs which lay the foundation for more sophisticated mathematical thinking. Students' abilities to interpret, analyze, retrieve and use different mathematics concepts become crucial for future work in science. To improve students' performance, reduce high failure rates, and to create long-term sustainable teaching and learning strategies for large mathematics classes, MyMathLab was used for teaching the Algebra and Trigonometry course at one comprehensive Canadian University. MyMathLab was used during two classes in consecutive semesters: first with a class of 26 and then with a class of 127 students. In addition to the face-to-face teaching format, written assignments, midterm and final examinations, students were required to complete 10 MyMathLab quizzes each semester. A variety of factors contribute to students' achievements, thus making comparisons difficult, but based on the assessment scores taken prior to enrolment in the course and on the average final grade scores it is believed that the group using MyMathLab made greater progress than students who did not (Kondratieva & Radu, 2008). A snapshot of the research study testing the effectiveness of this environment shows that the percentage of students who received A*s* increased from 12.9 percent at the time when the course was taught without MyMathLab to 15.4 percent at the time when MyMathLab was incorporated in the course. Furthermore, the percentage of students receiving B*s* increased from 19.3 percent to 26.9 percent (Kondratieva & Radu, 2008).

Subsequently, MyMathLab was used within a blended learning environment, where the instructor combined traditional teaching methods and computer-mediated instruction strategies as part of classroom teaching. The instructor had the opportunity to monitor, control, adjust and match the blended teaching process with the weekly online homework offered through the online laboratory created to assist students in their mathematics e-journey. Again, students were required to complete 10 MyMathLab quizzes. While it is apparent that students' performances improve (Kondratieva & Radu, 2008), it makes sense to analyze students' emotional structures in blended and face-to-face environments, because how much success they eventually have in mathematics is

intimately related to both the cognitive and the affective processes that characterize their thinking and problem solving in the subject (McLeod & Adams, 1989; Goldin, 2008). If the emotional tone of mathematical learning is integrally related to how mathematical information is perceived, processed, stored or retrieved, the potential value of studying the impacts of this learning could be essential. And this leads to one significant research question: Do students' mathematical intimacy and its positive by-products, namely confidence and persistence vary within blended and face-to-face environments?

In recent years, mathematics education researchers have started to pay attention to the role affective elements play in doing mathematics. Largely portrayed as encompassing emotions, beliefs, attitudes and values/morals/ethics, the affective domain is of primary concern for mathematicians and mathematics educators since it plays a fundamental role in the development and long-term appreciation of mathematics knowledge. Mathematics education researchers concerned with the learning of mathematics highlighted the importance of emotions in learning and problem solving performance (McLeod & Adams, 1989; DeBellis & Goldin, 1997; DeBellis, 1999). The emotive aspects of knowing could influence one's acquisition of mathematics knowledge. Affect, viewed as a representational system interacts with the cognitive representation systems, such as verbal, imagistic, formal notational and executive control (Goldin, 1987; 1988). As a representational system, affect includes changing states of emotional feelings during mathematics problem solving, also known as local affect, as well as more permanent and stable constructs, known as global affect (DeBellis & Goldin, 2006). Situated within the context of local affect, mathematical intimacy is an affective structure that carries emotional meaning and weight for students.

Based on the psychological research, mathematical intimacy is defined as a form of intimacy that consists of two components: intimate interactions and intimate relationships (Prager, 1995; DeBellis, 1998). A series of intimate mathematical interactions build up intimate relationships. Thus, the core of this affective structure lies with the intimate interactions, which are characterized by intimate mathematical behaviours and intimate mathematical experiences. According to DeBellis (1998), examples of intimate mathematical behaviours include "the distance a problem solver places between himself and his work, cradling his work, temporary loss of hearing external noises because he is so focused and consumed by the interaction, and hesitation in sharing mathematical solutions"; and examples of intimate mathematical experiences include warmth, passion, time suspension, vulnerability, loyalty, and positive emotions such as joy, excitement, affection, or amusement. But, beyond this organized structure, experiencing mathematical intimacy is equivalent to being highly engaged in problem solving, having a warm-hearted dialogue with various math concepts, analyzing and comprehending its most inner structures, creating a close bond with mathematics. Moreover, Goldin (2008) claims that mathematical engagement; a form of mathematical intimacy may be connected to flow. This connection is based on items, such as loss of self-consciousness while being highly engaged in problem solving, altered perception of time and experiencing satisfaction. But, in addition to these associations, the mathematical intimacy and flow analogy could include the challenge-skill balance, clear goals and intense concentration. Experiencing mathematical intimacy invokes a challenge-skill balance since otherwise, assuming conditions of anxiety or boredom mathematical intimacy could not come to fruition. Becoming intimately engaged in solving math problems implies one possible clear goal of solving the problem, and furthermore assumes a certain level of concentration. Enjoyment, an essential component of mathematical intimacy, is well related to flow in doing mathematics (Seifert, Radu & Doyle, 2009). Mathematical intimacy may lead to positive outcomes such as confidence in personal abilities to continue future problem solving activities, perseverance in pursuing solving math problems, or willingness to take risks due to a sense of safety provided by mathematical intimacy; or negative outcomes, such as frustration, disappointment, or anger due to unexpected outcomes while solving math problems (DeBellis, 1998).

Method

Participants in this study came from three classes of students enrolled in a first year mathematics course on Algebra and Trigonometry. The course is a prerequisite for Calculus courses, and students who take the course have failed to achieve the cut-off score needed to enrol in Calculus on a mathematics skills screening test. Two classes were offered in lecture-only format. Of the 69 students in the first class, 40 agreed to participate. Of the 66 students in the second class, 41 agreed to complete the survey. The third class was comprised of students enrolled in a blended version of the course, which combined lectures with participation in MyMathLab. Of the 72 students in this class, 29 agreed to participate. In total, data from 108 students were included in the analyses; 2 students were excluded because of missing data.

The course curriculum includes sections on real numbers, functions (e.g. exponential, logarithmic), trigonometry, analytic trigonometry, and polynomials. The standard course layout involves four hours of face-to-face lectures per week, written homework, midterms and final exams, and no computer mediated teaching and learning. The instructor of the blended class used a combination of online and face-to-face teaching methods in four hour time frame per week. In the first two weeks of classes, computer laboratories were held and the instructor guided students throughout the MyMathLab registration process and explained the features of the software. Students were expected to complete their weekly quizzes via MyMathLab's online setting. Unlike the previous semesters when MyMathLab was implemented (Kondratieva & Radu, 2008) and computer laboratories were in place and students attended them on a weekly basis; this semester the lab instructor went to class once a week for about 20 minutes. During that time, the lab instructor clarified the math examples where most common mistakes occurred in the previous week's e-homework and used MyMathLab help files that show how to get the solution of a problem correctly. Thus, the objective was to show students how they could get help from the software when they worked on their MyMathLab homework and studied for tests.

Early in the semester, students in both groups (lecture-only and blended format) completed a 64-item survey assessing several constructs related to affect and flow in mathematics. Some items of the survey used in the confidence, enjoyment and interest factors were adapted from the research work of Galbraith and Haines (1998), Tapia and Marsh (2004) and Cretchley (2008). For the purposes of this study, only those items assessing mathematical intimacy, confidence, persistence and flow were used. The intimacy construct was operationalized using items that asked students about feelings they have when doing mathematics (see Figure 1). The items were in 4-point Likert format (strongly disagree, disagree, agree or strongly agree), and were subsequently dichotomized (disagree or agree) for analyses. These variables were then subjected to a factor analysis using MPlus (Muthén & Muthén, 2006; see Figure 1). The model fit the data well: χ^2 (22)=26.38, p=.24; CFI=.993; TLI=.994; RMSEA=.043. A multi-group analysis indicated that the mean and variance of the mathematical intimacy factor did not differ between the lecture-only and blended classes.

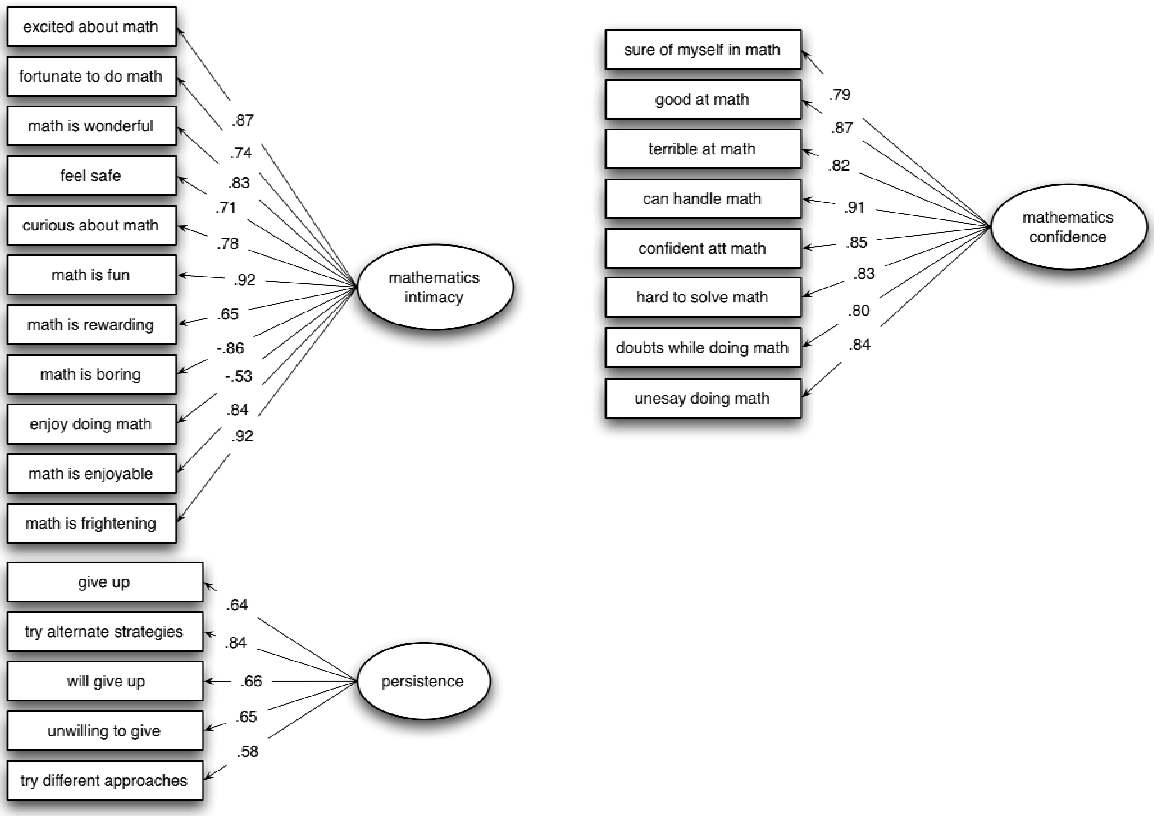

Figure 1. Factor structure for mathematical intimacy, confidence and persistence. Coefficients are standardized regression coefficients using probit regression.

The confidence construct was operationalized using eight 4-point Likert items that asked students about their perceived ability to solve mathematics problems (see Figure 1). A multi-group analysis indicated that the means

and variances were equal across the two groups (χ^2 (19)=9.930, p=.45; CFI=1.0; TLI=1.0; RMSEA=.00). The persistence construct (five items in 4 point Likert format; Figure 1) also displayed equal means and variances across the two groups (χ^2 (5)=3.324, p=.65; CFI=1.0; TLI=1.0; RMSEA=.00).

Flow was assessed using a nine-item scale similar to the Short Flow Scale (Jackson, Martin & Eklund, 2008). Each item, taken from the Dispositional Flow Scale-2 (Jackson & Eklund, 2004), assessed one of the nine dimensions of the flow construct (Csikzsentmihalyi, 1990) using a 5 point Likert format; items were summed to establish a total flow score. A multi-group analysis suggested that the variance of the flow variable was not statistically different between the two groups, but the means were. Students in the blended classroom reported lower flow scores for mathematics than those in the lecture-only group, and the difference was substantial (ES=.62).

At the end of the semester, those students in the blended classroom completed the survey a second time, but with some modifications. Rather than asking about experiences with mathematics, the items asked about experiences in doing mathematics using MyMathLab. The factor structures for mathematical intimacy, competence and persistence obtained in the analysis of the first survey was then imposed on the time 1 and time 2 responses to relevant items of students in the blended classroom.

Results

The mathematical intimacy factor structure from survey one was imposed upon the data for the students in the blended classroom, and a multi-group analysis performed. The imposed factor structure fit the data well under both conditions of equal means and unequal variances, and unequal means and equal variances. The results of the multi-group analysis suggested that students in the blended classroom had slightly lower intimacy scores for their MyMathLab experiences than for mathematics in general, but the differences were not statistically detectable. However, the variance of the mathematical intimacy factor was larger for their MyMathLab experiences than for mathematics in general. This is evident in Figure 2. Mathematical intimacy scores for mathematics in general were clustered around the mean. However, there was much greater variability and range in their mathematical intimacy scores for MyMathLab. One-third of the students had mathematical intimacy scores for MyMathLab that were well below the minimum value for mathematics in general. On the other hand, one-quarter of the students had mathematics scores for MyMathLab experiences that were well above the maximum of mathematics in general. This suggests that participation in MyMathLab lead students to either to turn their backs in resentment of mathematics or to have a greater affinity towards mathematics.

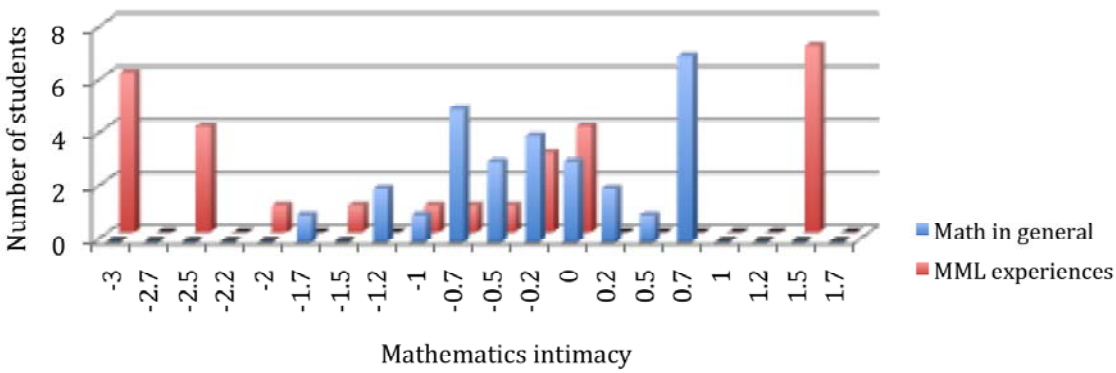

Figure 2. Distribution of mathematical intimacy scores for students in the blended learning group.

A similar analysis was conducted for the confidence items with parallel result to the analysis of the intimacy items. The factor structure from the first analysis was imposed on the time 1 and time 2 item responses of students in the blended class. Although the mean score for MyMathLab experience was similar to mathematics in general, the variance of the MyMathLab experience factor was greater than for mathematics in general. This was the result of a slight shift upwards for some students, accompanied by a lowering of confidence for others.

Specifically, 34% of the MyMathLab confidence scores were lower then the minimum mathematics confidence score. However, 21% of the MyMathLab confidence scores were higher than the maximum mathematics confidence score. In other words, participation in MyMathLab tended to either boost students' confidence or lower it.

Persistence items were analyzed in a similar manner to confidence and intimacy items. However, there were no differences in either the mean and variance of the persistence factor. This suggests that participation in MyMathLab had no discernible overall effect on students' willingness to exert effort to solve problems. Finally, the results showed that students' level of flow did not vary between MyMathLab and math in general. The multi-group analysis indicated that the mean and variance was not different across contexts.

Discussion

In this study, students in a blended learning environment responded to two sets of items assessing mathematical intimacy, confidence, persistence and flow. The first set of items asked about mathematics in general; the second asked students about these constructs in the context of MyMathLab. Students in the blended class had mathematical intimacy scores similar to students in the lecture group. However, the mathematical intimacy scores for many students in the blended class were lower for MyMathLab than for mathematics in general, as indicated by an increase in range and variance. A similar finding was found for confidence, but not persistence and flow.

Slightly lower mathematical intimacy scores for many students in the MyMathLab framework might interpreted as a result of their inability of creating a close bond and in experiencing joy and excitement in doing mathematics in the brief period of time when they were solving math online. Mathematical intimacy may foster the appearance of positive outcomes, such as confidence and perseverance (DeBellis, 1998). As such, for many students in the blended class, lower scores of mathematical intimacy translate into a decreased enjoyment and sense of well being that subsequently leads to poorer confidence, and thus slightly lowered confidence scores. But, persistence is also viewed as a possible outcome of mathematical intimacy (DeBellis, 1998). However, similar scores in persistence between the two groups might be explained through students' devotion in passing the course or in obtaining a good grade regardless of mathematical context: blended or traditional. Based on the theoretical similarities of the mathematical intimacy and flow (Goldin, 2008), we expected that students would have similar experiences and scores. It came as a surprise that even if mathematical intimacy scores were comparable in the blended and lecture-format settings with respect to general math, there was a substantial statistical difference in flow scores, as the students in the blended classroom reported lower flow scores in general math than those in the lecture only class setting.

There are a number of limitations to this study that suggest further investigation is warranted. First, the measures may not have been in sufficient temporal proximity to accurately measure students' affect. For example, the flow items asked about flow characteristics in general, and not about a specific activity at a given moment, and not capturing students' inner experiences (Hurlburt & Heavey, 2006). Second, the study may have been limited by low statistical power to detect differences. The small sample size and the lack of a true pre-test post-test design reduced statistical power to detect differences.

However, the study suggests that MyMathLab may have a positive effect for some students, and a negative effect for others. A mixed-methods study employed pre-test post-test design with either case studies or interviews could shed much more light on students' inner experiences when using online technology for learning mathematics.

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